The ambiguities of dimensional regularization scheme

1975 J. Phys. A: Math. Gen. 8 334

(http://iopscience.iop.org/0305-4470/8/3/007)

J. Phys. A: Math. Gen., Vol. 8, No. 3, 1975. Printed in Great Britain. © 1975

The ambiguities of dimensional regularization scheme

M Nouri-Moghadam and J G Taylor
Department of Mathematics, King's College, London WC2R 2LS, UK

Received 1 July 1974

Abstract. The dimensional regularization technique is used to evaluate one-loop corrections to gravitational self-interaction. The application of such a scheme contains certain ambiguities which are presented and discussed.

Attempts to remove these ambiguities have not yet proved successful.

1. Introduction

It has been suggested by several authors that analytic continuation in the number of space dimensions may be a convenient regularizing technique especially in the case of gauge theories (Arnowitt and Deser 1959, Arnowitt *et al* 1959, 1960, Feynman 1963, Mandelstam 1968a, b) where it is essential that the regularization scheme respects the Ward–Slavnov identities.

Dimensional regularization ('t Hooft and Veltman 1972, Ashmore 1972, 1973) has been successful in particular for gravitational interactions. In this case elimination of infrared divergences, arising from massless tadpoles of the form $\int d^{2\omega}q(q^2)^{-1}$ (2ω = total number of space–time), and also evaluation of $\delta^4_{(0)}$ (Capper and Leibbrandt 1973) lead to calculation of one-loop graviton and matter self-energies which satisfy the Ward–Slavnov identities.

The aim of this paper is to point out the ambiguities of the scheme employed for evaluating one-closed-loop corrections.

Dimensional regularization may be summarized briefly as follows. First each momentum-space integral is defined over a 2ω-dimensional Euclidean space (ω complex), and each integral for *general* ω is evaluated. Then the resulting expression is expanded in a Laurent series about the pole $\omega = 2$ (ie, four-dimensional space–time). Pole terms in the Laurent expression may cancel by inserting appropriate counter terms in the interaction Lagrangian. The value of each integral is given by the remaining part of the expansion continued to Minkowski space. The total amplitude for graviton–matter self-energy and its resultant connected Green function $Q_{\alpha\beta\mu\lambda}$ is expanded about $\omega = 2$ and is continued analytically to Minkowski space leading to a decomposition of $Q_{\alpha\beta\mu\lambda}$ into a pole term and the finite (physical) term.

However in continuing integrals to 2ω dimensions they could be multiplied by any arbitrary function $f_i(\omega)$ which is a real analytic function of ω with $f_i(2) = 1$. Then the infinite part of $Q_{\alpha\beta\mu\lambda}$ and the Lagrangian counter term (ΔL) remain unchanged but their finite part contains arbitrary parameters $f_i'(2)$ which are ambiguous.

In § 2 we outline briefly some standard results and identities which are needed for our calculation. In § 3 we prove the new self-energy loop contributions satisfy the Ward–Slavnov identities. Finally in § 4 we summarize the result.

2. Some standard results and identities

Since there have been some mistakes and misprints in previous papers, in particular Capper *et al* (1973), we must first summarize some standard results. This will be more suitable than spelling out alternatives in detail. It will also enable us to set up the problem more fully.

We start with the simple case of gravitation alone.

$$L = +\frac{2}{K^2}\sqrt{(-g)}g^{\mu\nu}R_{\mu\nu} = \frac{2}{K^2}\sqrt{(-g)}R \tag{2.1}$$

where $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ the curvature tensor and R the curvature scalar.

This Lagrangian can be written in the form

$$L = \frac{1}{2K^2}\left[\mathfrak{g}^{\rho\sigma}\mathfrak{g}_{\lambda\mu}\mathfrak{g}_{\beta\nu} - \frac{1}{n-2}\mathfrak{g}^{\rho\sigma}\mathfrak{g}_{\mu\beta}\mathfrak{g}_{\lambda\nu} - 2\delta^{\sigma}_{\beta}\delta^{\rho}_{\lambda}\mathfrak{g}_{\mu\nu}\right]\mathfrak{g}^{\mu\alpha}{}_{,\beta}\mathfrak{g}^{\lambda\nu}{}_{,\sigma} \tag{2.2}$$

where $\mathfrak{g}^{\mu\nu} = (-g)^{1/2}g^{\mu\nu}$ and n is the dimension of the space. The generating functional in this case is (Fradkin and Tyutin 1970)

$$Z[j_{\mu\nu}] = \int \mathrm{d}[\mathfrak{g}^{\mu\nu}]\Delta[\mathfrak{g}^{\mu\nu}]\exp\left[\mathrm{i}\int \mathrm{d}x\left(L + \frac{1}{K}g^{\mu\nu}j_{\mu\nu} - \frac{1}{K^2}(\partial_\mu\mathfrak{g}^{\mu\nu})^2\right)\right] \tag{2.3}$$

where $\Delta[\mathfrak{g}^{\mu\nu}]$ is the fictitious particle contribution and $\rho[B] = -(K^2\alpha)^{-1}(\partial_\mu\mathfrak{g}^{\mu\nu})^2$, the gauge braking term, is known as the weight function. The resultant S-matrix is unitary and independent of choice of gauge (parameter α) (for simplicity fix the gauge by choosing $\alpha = -1$).

We use the standard Minkowski space based approach to covariant quantization in which the quantized metric $\mathfrak{g}_{\mu\nu}$ is decomposed as

$$\mathfrak{g}^{\mu\nu} = \delta^{\mu\nu} + K\phi^{\mu\nu} \tag{2.4}$$

where $\delta^{\mu\nu}$ is the n-dimensional Kronecker delta ($\delta^{\mu\mu} = n$). Then

$$\mathfrak{g}_{\mu\nu} = \delta_{\mu\nu} - K\phi_{\mu\nu} + K^2\phi_{\mu\nu} + K^2\phi_{\mu\alpha}\phi_{\alpha\nu} - K^3\phi_{\mu\alpha}\phi_{\alpha\beta}\phi_{\mu\nu} + \mathrm{O}(K^4) \tag{2.5}$$

and there is no need to distinguish between upper and lower indices of $\phi_{\mu\nu}$.

Writing the Lagrangian L as

$$L = \sum_{j=2}^{\infty} K^{j-2}L(j) \tag{2.6}$$

then

$$L_{(2)}(x) = \tfrac{1}{2}\partial_\mu\phi_{\nu\lambda}\partial_\mu\phi_{\nu\lambda}(n) - \frac{1}{2(n-2)}\partial_\mu\phi_{\nu\nu}\partial_\mu\phi_{\rho\rho}(n) - \partial_\mu\phi_{\mu\nu}\partial_\rho\phi_{\rho\nu}(n). \tag{2.7}$$

The free graviton propagator will be

$$D_{\alpha\beta,\lambda\mu}(x) = \tfrac{1}{2}[\delta_{\alpha\lambda}\delta_{\beta\mu} + \delta_{\alpha\mu}\delta_{\beta\lambda} - \delta_{\alpha\beta}\delta_{\lambda\mu}]D(n) \tag{2.8a}$$

$$D(n) = \Gamma(\omega-1)(-x^2)^{1-\omega}/4\pi^{\omega}$$

where $D(n) = (4\pi x^2)^{1-n/2}$ is the massless scalar propagator. Alternatively in momentum–space

$$D_{\alpha\beta,\lambda\mu}(p) = \frac{1}{2p^2}(\delta_{\alpha\lambda}\delta_{\beta\mu} + \delta_{\alpha\mu}\delta_{\beta\lambda} - \delta_{\alpha\beta}\delta_{\lambda\mu}). \tag{2.8b}$$

The propagator of the fictitious particle is obtained from $\Delta[\tilde{g}^{\mu\nu}]^{-1}$ which is

$$D_{\alpha\beta}(p) = \frac{\delta_{\alpha\beta}}{p^2}. \tag{2.9}$$

To obtain vertex functions in momentum–space we need $L_{(3)}$ which is

$$L_{(3)} = \frac{1}{2}\left[Q_{\sigma\rho}\left(Q_{\mu\kappa,\rho}\phi_{\mu\kappa,\sigma} - \frac{1}{n-2}Q_{\mu\mu,\rho}Q_{\nu\nu,\sigma}\right)\right.$$
$$\left. + \phi_{\mu\tau}\left(\phi_{\mu\sigma,\tau}\phi_{\lambda\tau,\sigma} - \phi_{\mu\kappa,\rho}\phi_{\tau\kappa,\rho} + \frac{1}{n-2}\phi_{\mu\tau,\rho}\phi_{\nu\nu,\rho}\right)\right] \tag{2.10}$$

which gives the graviton–ζ–η vertex (the two different fictitious particles are ζ, η, see figure 2) and the three-graviton vertex (see figure 1) respectively as

$$V_{\alpha\beta,\lambda,\mu}(k_1, k_2, k_3) = -\delta_{\lambda(\alpha}k_{1\beta)}k_{2\mu} + \delta_{\lambda\mu}k_{2(\alpha}k_{3\beta)} \tag{2.11}$$

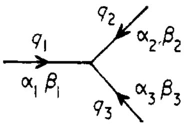

Figure 1. The three-graviton vertex.

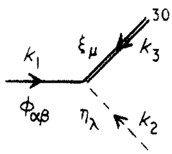

Figure 2. The two fictitious particle graviton vertex.

and

$$V(q_1, q_2, q_3)_{\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3}$$
$$= -\frac{1}{2}\left[q^2_{(\alpha_1}q^3_{\beta_1)}\left(2\delta_{\alpha_2(\alpha_3}\delta_{\beta_3)\beta_2} - \frac{2}{n-2}\delta_{\alpha_2\beta_2}\delta_{\alpha_3\beta_3}\right)\right.$$
$$+ q^1_{(\alpha_2}q^3_{\beta_2)}\left(2\delta_{\alpha_1(\alpha_2}\delta_{\beta_3)\beta_1} - \frac{2}{n-2}\delta_{\alpha_1\beta_1}\delta_{\alpha_3\beta_3}\right)$$
$$+ q^1_{(\alpha_3}q^2_{\beta_3)}\left(2\delta_{\alpha_1(\alpha_2}\delta_{\beta_2)\beta_1} - \frac{2}{n-2}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}\right)$$
$$+ 2q^3_{(\alpha_2}\delta_{\beta_2)(\alpha_1}\delta_{\beta_1)(\alpha_3}q^2_{\beta_3)} + 2q^1_{(\alpha_3}\delta_{\beta_3)(\alpha_2}\delta_{\beta_2)(\alpha_1}q^3_{\beta_1)}$$
$$+ q^2q^3\left(\frac{2}{n-2}\delta_{\alpha_1(\alpha_3}\delta_{\beta_3)\beta_1}\delta_{\alpha_2\beta_2} - \delta_{\alpha_1(\alpha_2}\delta_{\beta_2)(\alpha_3}\delta_{\beta_3)\beta_1} + \frac{2}{n-2}\delta_{\alpha_1(\alpha_2}\delta_{\beta_2)\beta_1}\delta_{\alpha_3\beta_3}\right)$$
$$+ 2q^2_{(\alpha_1}\delta_{\beta_1)(\alpha_3}\delta_{\beta_3)(\alpha_2}q^1_{\beta_2)}$$
$$+ q^1_1q^3\left(\frac{2}{n-2}\delta_{\alpha_2(\alpha_1}\delta_{\beta_1)\beta_2}\delta_{\alpha_3\beta_3} + \frac{2}{n-2}\delta_{\alpha_2(\alpha_3}\delta_{\beta_3)\beta_2}\delta_{\alpha_1\beta_1} - 2\delta_{\alpha_2(\alpha_1}\delta_{\beta_1)(\alpha_3}\delta_{\beta_3)\beta_2}\right)$$
$$\left. + q^1_0q^2\left(\frac{2}{n-2}\delta_{\alpha_3(\alpha_1}\delta_{\beta_1)\beta_3}\delta_{\alpha_2\beta_2} + \frac{2}{n-2}\delta_{\alpha_3(\alpha_2}\delta_{\beta_2)\beta_3} + \delta_{\alpha_1\beta_1} - 2\delta_{\alpha_3(\alpha_1}\delta_{\beta_1)(\alpha_2}\delta_{\beta_2)\beta_3}\right)\right] \tag{2.12}$$

where

$$A_{(\alpha}B_{\beta)} \equiv \tfrac{1}{2}(A_\alpha B_\beta + A_\beta B_\alpha).$$

All the possible one-closed-loop graphs are shown in figure 3. The contributions from figures 3(*a*) and (*b*) have been explicitly calculated, while the massless tadpole diagram (*c*) is consistently equated to zero (Capper and Leibbrandt 1972). Similar restriction is applied to (*f*), which corresponds to the $\delta^4_{(0)}$ term (Capper and Leibbrandt 1972, 1973). However, (*d*) and (*e*) containing zero-momentum propagator of mass zero have not been handled. Self-energy contributions from diagrams (*a*) and (*b*) are denoted by $F_{\alpha\beta\alpha'\beta'}(p)$ and $R_{\alpha\beta\alpha'\beta'}(p)$ respectively.

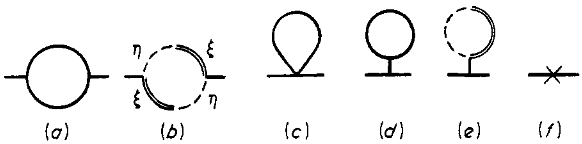

Figure 3. Lowest-order contribution to the graviton self-energy.

2.1. The fictitious particle self-energy loop

The contribution from fictitious loop is given by (figure 4)

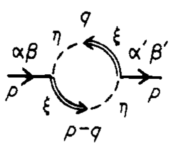

Figure 4. Fictitious particle loop.

$$F_{\alpha\beta\alpha'\beta'}(p) = -K^2 \int d^{2\omega}q V_{\alpha\beta,\lambda,\sigma}(p, -p, q-p)D_{\lambda\lambda'}(-q)V_{\alpha'\beta',\sigma',\lambda'}(-p, p-q, q)xD_{\sigma\sigma'}(q-p)$$

$$= -K^2\int d^{2\omega}q[q^2(q-p)^2]^{-1}[p_\alpha p_\beta p_{\alpha'}q_{\beta'} + q_\alpha p_\beta p_{\alpha'}p_{\beta'} - q_\alpha q_\beta p_{\alpha'}p_{\beta'}$$

$$+2\omega q_\alpha q_\beta q_{\alpha'}q_{\beta'} + (1-2\omega)q_\alpha q_\beta q_{\alpha'}p_{\beta'} - (1+2\omega)p_\alpha q_\beta q_{\alpha'}q_{\beta'}$$

$$+(2\omega-1)p_\alpha q_\beta p_{\alpha'}q_{\beta'}]. \qquad (2.13)$$

To evaluate various integrals in this expression *basic integral* I_1 is used,

$$I_1 = \int d^{2\omega}q[q^2(q-p)^2]^{-1} \qquad (2.14)$$

$$= \pi^\omega[\Gamma(2\omega-2)]^{-1}\Gamma(2-\omega)\Gamma(\omega-1)\Gamma(\omega-1)(p^2)^{\omega-2} \qquad (2.15)$$

where the following formulae have been used, namely:

$$(q^2)^{-1} = \int_0^\infty \exp(-\alpha q^2)\, d\alpha \qquad q^2 > 0 \qquad (2.16)$$

$$\int d^{2\omega}q \exp(-aq^2 + 2b\,.\,q) = \left(\frac{\pi}{a}\right)^\omega \exp\left(\frac{b^2}{a}\right) \qquad a > 0 \qquad (2.17)$$

where

$$\Gamma(z) = \int_0^\infty dt t^{z-1}\exp(-t) \qquad \mathrm{Re}(z) > 0. \qquad (2.18)$$

The various gamma functions in (2.15) are continued analytically to other values of ω other than $\omega = 2$ by means of the partial fraction expansion

$$\Gamma(1-\omega) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1-\omega)} + \int_1^{\infty} \mathrm{d}t\, t^{-\omega} \mathrm{e}^{-t}. \tag{2.19}$$

The concept of 'analytic continuation in the number of dimensions' is the most important single feature in the technique of dimensional regularization.

The various integrals in equation (2.13) can be obtained by differentiation of equation (2.17) with respect to b_μ (b_μ vector over a space of 2ω dimensions). Thus:

$$\int \mathrm{d}^{2\omega}q[q^2(q-p)^2]^{-1} q_\alpha = p_\alpha I_2 \tag{2.20}$$

$$\int \mathrm{d}^{2\omega}q[q^2(q-p)^2]^{-1} q_\alpha q_\beta = \delta_{\alpha\beta} I_3 + p_\alpha p_\beta I_4 \tag{2.21}$$

$$\int \mathrm{d}^{2\omega}q[q^2(q-p)^2]^{-1} q_\alpha q_\beta q_\gamma = p_\alpha p_\beta p_\gamma I_5 + E_{\alpha\beta\gamma} I_6 \tag{2.22}$$

$$\int \mathrm{d}^{2\omega}q[q^2(q-p)^2]^{-1} q_\alpha q_\beta q_\gamma q_\sigma = p_\alpha p_\beta p_\gamma p_\sigma I_7 + G_{\alpha\beta\gamma\sigma} I_8 + H_{\alpha\beta\gamma\sigma} I_9 \tag{2.23}$$

where

$$E_{\alpha\beta\gamma} \equiv \delta_{\alpha\beta} p_\gamma + \delta_{\beta\gamma} p_\alpha + \delta_{\gamma\alpha} p_\beta \tag{2.24a}$$

$$G_{\alpha\beta\gamma\sigma} \equiv \delta_{\alpha\beta} p_\gamma p_\sigma + \delta_{\beta\gamma} p_\alpha p_\sigma + \delta_{\beta\sigma} p_\alpha p_\gamma + \delta_{\alpha\gamma} p_\beta p_\sigma + \delta_{\alpha\sigma} p_\beta p_\gamma + \delta_{\gamma\sigma} p_\alpha p_\beta \tag{2.24b}$$

$$H_{\alpha\beta\gamma\sigma} \equiv \delta_{\alpha\beta}\delta_{\gamma\sigma} + \delta_{\alpha\sigma}\delta_{\beta\gamma} + \delta_{\beta\sigma}\delta_{\alpha\gamma} \tag{2.24c}$$

and each of integrals $I_2, \ldots, I_9$ is obtained in terms of I_1 as

$$I_2 = (2)^{-1} I_1 \tag{2.25}$$

$$I_3 = -[4(2\omega-1)]^{-1} I_1 \tag{2.26}$$

$$I_4 = \omega[2(2\omega-1)]^{-1} I_1 \tag{2.27}$$

$$I_5 = (\omega+1)[4(2\omega-1)]^{-1} I_1 \tag{2.28}$$

$$I_6 = -[8(2\omega-1)]^{-1} p^2 I_1 \tag{2.29}$$

$$I_7 = (\omega+1)(\omega+2)[4(4\omega^2-1)]^{-1} I_1 \tag{2.30}$$

$$I_8 = -(\omega+1)[8(4\omega^2-1)]^{-1} p^2 I_1 \tag{2.31}$$

$$I_9 = [16(4\omega^2-1)]^{-1} (p^2)^2 I_1. \tag{2.32}$$

Substituting each integral into equation (2.13) gives

$$\begin{aligned} F_{\alpha\beta\alpha'\beta'}(p) = -K^2[&p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_2 + (2\omega-1)(p_\alpha p_{\alpha'} \delta_{\beta\beta'} I_3 + p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_4) \\ &-(2\omega+1)(p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_5 + p_{(\alpha} F_{\beta)\alpha'\beta'} I_6) \\ &-(2\omega-1)(p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_5 + p_{(\alpha'} F_{\beta')\alpha\beta} I_6) - p_{\alpha'} p_{\beta'}(\delta_{\alpha\beta} I_3 + p_\alpha p_\beta I_4) \\ &+2\omega(p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_7 + G_{\alpha\beta\alpha'\beta'} I_8 + H_{\alpha\beta\alpha'\beta'} I_9) + p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_2] \end{aligned} \tag{2.33}$$

and finally in terms of I_1 we get

$$F_{\alpha\beta\alpha'\beta'}(p) = +K^2[p_\alpha p_\beta p_{\alpha'} p_{\beta'} F_1(p^2) + \delta_{\alpha\beta}\delta_{\alpha'\beta'}F_2(p^2)$$
$$+(\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \delta_{\beta\alpha'}\delta_{\alpha\beta'})F_3(p^2) + (\delta_{\alpha\beta}p_{\alpha'}p_{\beta'} + \delta_{\alpha'\beta'}p_\alpha p_\beta)F_4(p^2)$$
$$+(\delta_{\alpha\alpha'}p_\beta p_{\beta'} + \delta_{\beta\alpha}p_\alpha p_{\beta'} + \delta_{\alpha\beta'}p_\beta p_{\alpha'} + \delta_{\beta\beta'}p_\alpha p_{\alpha'}) \times F_5(p^2)] \tag{2.34}$$

where

$$F_1 = -[2(4\omega^2-1)]^{-1}(\omega^3+3\omega^2-2\omega-2)I_1 \tag{2.35a}$$
$$F_2 = F_3 = -[8(4\omega^2-1)]^{-1}\omega(p^2)^2 I_1 \tag{2.35b}$$
$$F_4 = -8[8(4\omega^2-1)]^{-1}(2\omega^2+2\omega+1)p^2 I_1 \tag{2.35c}$$
$$F_5 = -16[16(4\omega^2-1)]^{-1}p^2 I_1. \tag{2.35d}$$

The explicit form of these F will be required to verify the Slavnov–Ward identities.

2.2. *The graviton self-energy loop*

Similar procedure may be applied to the graviton loop shown in figure 5. The self-energy amplitude is given by

$$R_{\alpha\beta\alpha'\beta'}(p) = \frac{K^2}{32}\int \frac{d^{2\omega}q}{q^2(p-q^2)}\Big\{16[4p_\alpha p_\beta p_{\alpha'}p_{\beta'} - 2(p_\alpha p_\beta p_{(\alpha'}q_{\beta')} + q_{(\alpha}p_{\beta)}p_{\alpha'}p_{\beta'})$$
$$+(2\omega+3)(p_\alpha p_\beta q_{\alpha'}q_{\beta'} + q_\alpha q_\beta p_{\alpha'}p_{\beta'}) - \omega(2\omega+1)(p_{(\alpha}q_{\beta)}q_{\alpha'}q_{\beta'} + q_\alpha q_\beta q_{(\alpha'}p_{\beta')})$$
$$+(2\omega^2-3\omega-2)p_{(\alpha}q_{\beta)}p_{(\alpha'}q_{\beta')} + (2\omega+1)\omega q_\alpha q_\beta q_{\alpha'}q_{\beta'}]$$
$$+16\{p^2[-(2\omega+4)p_{(\alpha}\delta_{\beta)(\alpha'}p_{\beta')} - 2\omega p_{(\alpha}\delta_{\beta)(\alpha'}q_{\beta')} - 2\omega q_{(\alpha}\delta_{\beta)(\alpha'}p_{\beta')}$$
$$+4\omega q_{(\alpha}\delta_{\beta)(\alpha'}q_{\beta')}] + 4(\omega+1)p\,.\,q\,.\,p_{(\alpha}\delta_{\beta)(\alpha'}p_{\beta')} - 4(\omega+1)q^2 p_{(\alpha}\delta_{\beta)(\alpha'}p_{\beta')}\}$$
$$+\frac{8}{\omega-1}\delta_{\alpha(\alpha'}\delta_{\beta')\beta}[(q^2)^2(4\omega^2-4\omega-2) + q^2\,.\,q\,.\,q(-8\omega^2+8\omega+4)$$
$$+q^2p^2(8\omega^2+4\omega-22) + p^2p\,.\,q(-8\omega^2+12\omega-8)$$
$$+(p\,.\,q)^2 - 20\omega + 28) + (p^2)^2(4\omega^2-5)]$$
$$+\frac{4}{\omega-1}\delta_{\alpha\beta}\delta_{\alpha'\beta'}[(q^2)^2(4\omega-2) - q^2\,.\,p\,.\,q(8\omega-4) - p^2q^2(6\omega-6)$$
$$+p^2p\,.\,q(10\omega-12) + 4(p\,.\,q)^2 + (p^2)^2(-\omega^2-6\omega+9)]$$
$$+\frac{8}{(\omega-1)^2}\Big[p^2\delta_{\alpha\beta}[4(\omega-1)p_{\alpha'}p_{\beta'} + (2\omega^3-3\omega^2-5\omega+6)p_{(\alpha'}q_{\beta')}$$
$$+(-2\omega^3+\omega^2+6\omega-5)q_{\alpha'}q_{\beta'}] + p\,.\,q\delta_{\alpha\beta}[-(2\omega-2)p_{\alpha'}p_{\beta'}$$
$$+(-4\omega^2+6\omega)(\omega-1)p_{(\alpha'}q_{\beta')} + (4\omega+2)(\omega-1)^2 q_{\alpha'}q_{\beta'}]$$
$$+q^2\delta_{\alpha\beta}[-(2\omega-3)(\omega-1)p_{\alpha'}p_{\beta'} + (4\omega+2)(\omega-1)^2 p_{(\alpha'}q_{\beta')}$$
$$+(-4\omega-2)(\omega-1)q_{\alpha'}q_{\beta'}] + \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \leftrightarrow \begin{pmatrix}a'\\ \beta'\end{pmatrix}\Big]\Big\}. \tag{2.36}$$

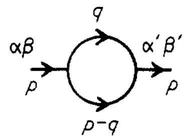

Figure 5. Graviton loop.

The final form of the graviton loop is

$$\begin{aligned} R_{\alpha\beta\alpha'\beta'}(p) = K^2[&p_\alpha p_\beta p_{\alpha'} p_{\beta'} R_1(p^2) + \delta_{\alpha\beta}\delta_{\alpha'\beta'} R_2(p^2) + (\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \delta_{\beta\alpha'}\delta_{\alpha\beta'})R_3(p^2) \\ &+ (\delta_{\alpha\beta}p_{\alpha'}p_{\beta'} + \delta_{\alpha'\beta'}p_\alpha p_\beta)R_4(p^2) + (\delta_{\alpha\alpha'}p_\beta p_{\beta'} + \delta_{\beta\alpha'}p_\alpha p_{\beta'} + \delta_{\alpha\beta'}p_\beta p_{\alpha'} \\ &+ \delta_{\beta\beta'}p_\alpha p_{\alpha'})R_5(p_2)] \end{aligned} \tag{2.37}$$

where

$$R_1 = [8(2\omega-1)]^{-1}(\omega^3-\omega^2+24\omega-8)I_1 \tag{2.38a}$$

$$R_2 = [32(\omega-1)^2(2\omega-1)]^{-1}(-7\omega^3+2\omega^2+13\omega)(p^2)^2 I_1 \tag{2.38b}$$

$$R_3 = [32(2\omega-1)]^{-1}(8\omega^2+5\omega-8)(p^2)^2 I_1 \tag{2.38c}$$

$$R_4 = [32(2\omega-1)(\omega-1)]^{-1}(2\omega^3-2\omega^2+20\omega+4)p^2 I_1 \tag{2.38d}$$

$$R_5 = [32(2\omega-1)]^{-1}(-8\omega^2-5\omega+10)p^2 I_1. \tag{2.38e}$$

The explicit forms of these R are essential for verifying the Slavnov–Ward identities.

2.3. *Total loop contribution*

Adding (2.33) and (2.36) the total contribution from the graviton and fictitious particle loops is given by

$$\begin{aligned} T_{\alpha\beta\alpha'\beta'}(p) = K^2[&p_\alpha p_\beta p_{\alpha'} p_{\beta'} T_1(p^2) + \delta_{\alpha\beta}\delta_{\alpha'\beta'} T_2(p^2) + (\delta_{\alpha\beta'}\delta_{\beta\beta} + \delta_{\beta\alpha'}\delta_{\alpha\beta'})T_3(p^2) \\ &+ (\delta_{\alpha\beta}p_{\alpha'}p_{\beta'} + \delta_{\alpha'\beta'}p_\alpha p_\beta)T_4(p^2) + (\delta_{\alpha\alpha'}p_\beta p_{\beta'} + \delta_{\beta\alpha'}p_\alpha p_{\beta'} + \delta_{\alpha\beta'}p_\beta p_{\alpha'} \\ &+ \delta_{\beta\beta'}p_\alpha p_{\alpha'})T_3(p^2)] \end{aligned} \tag{2.39}$$

where

$$T_1 = [8(4\omega^2-1)^{-1}(+2\omega^4-5\omega^3+35\omega^2+16\omega)I_1 \tag{2.40a}$$

$$T_2 = [32(\omega-1)^2(4\omega^2-1)]^{-1}(-14\omega^4-7\omega^3+36\omega^2+9\omega)(p^2)^2 I_1 \tag{2.40b}$$

$$T_3 = [32(4\omega^2-1)]^{-1}(16\omega^3+18\omega^2-15\omega-8)(p^2)^2 I_1 \tag{2.40c}$$

$$T_4 = [32(\omega-1)(4\omega^2-1)]^{-1}(+4\omega^4-10\omega^3+38\omega^2+32\omega+8)p^2 I_1 \tag{2.40d}$$

$$T_5 = [32(4\omega^2-1)]^{-1}(-16\omega^3-18\omega^2+15\omega+8)p^2 I_1. \tag{2.40e}$$

The Ward–Slavnov identities are derived (Capper and Ramon-Medrano 1973) from equation (2.2) which gives

$$2/\alpha\langle T\phi_{\mu\nu,\mu}(z)\phi_{\lambda\beta,\lambda}(y)\rangle = \delta_{\nu\beta}\delta(z-y) \qquad \alpha \neq 0 \tag{2.41}$$

which is true for each order of K and any value of α. Equation (2.41) implies for the lowest-order self-energy contribution $T_{\alpha\beta\alpha'\beta'}$ we have

$$p_\mu p_\nu D_{\mu\lambda,\gamma\beta}(p)T_{\gamma\beta\gamma'\beta'}(p)D_{\gamma'\beta',\nu\sigma}(p) = 0 \tag{2.42}$$

where $D(p)$ is graviton propagator. This identity in terms of connected Green function is

$$p_\mu p_\nu Q_{\nu\sigma,\mu\lambda}(p) = 0 \tag{2.43}$$

where

$$Q_{\nu\sigma,\mu\lambda}(p) = D_{\nu\sigma,\alpha\beta}(p)T_{\alpha\beta\alpha'\beta'}(p)D_{\alpha'\beta',\mu\lambda}(p). \tag{2.44}$$

The connected Green function $Q_{\alpha\beta\mu\lambda}$ can be obtained in terms of the T_i ($i = 1, \ldots, 5$) of equations (2.40*a*)–(2.40*e*)

$$Q_{\alpha\beta\mu\lambda}(p) = [4(p^2)^2]^{-1}[a_{1\alpha\beta\mu\lambda}T_1 + (\omega-1)^2 a_{2\alpha\beta\mu\lambda}T_2 + (a_{3\alpha\beta\mu\lambda} + (\omega-2)a_{2\alpha\beta\mu\lambda})T_3 + (\omega-1)a_{4\alpha\beta\mu\lambda}T_4 + a_{5\alpha\beta\mu\lambda}T_5] \tag{2.45}$$

with

$$a_{1\alpha\beta\mu\lambda} = 4p_\alpha p_\beta p_\mu p_\lambda - 2p^2\delta_{\mu\lambda}p_\alpha p_\beta - 2p^2\delta_{\alpha\beta}p_\mu p_\lambda + (p^2)^2\delta_{\alpha\beta}\delta_{\mu\lambda} \tag{2.46a}$$

$$a_{2\alpha\beta\mu\lambda} = 4\delta_{\alpha\beta}\delta_{\mu\lambda} \tag{2.46b}$$

$$a_{3\alpha\beta\mu\lambda} = 4(\delta_{\mu\lambda}\delta_{\alpha\beta} + \delta_{\alpha\lambda}\delta_{\beta\mu}) \tag{2.46c}$$

$$a_{4\alpha\beta\mu\lambda} = 4(p^2\delta_{\alpha\beta}\delta_{\mu\lambda} - \delta_{\mu\lambda}p_\alpha p_\beta - \delta_{\alpha\beta}p_\mu p_\lambda) \tag{2.46d}$$

$$a_{5\alpha\mu\lambda} = 4(\delta_{\mu\lambda}p_\alpha p_\beta + \delta_{\beta\mu}p_\alpha p_\lambda + \delta_{\alpha\lambda}p_\beta p_\mu + \delta_{\beta\lambda}p_\alpha p_\mu - 2\delta_{\alpha\beta}p_\mu p_\lambda - 2\delta_{\mu\lambda}p_\alpha p_\beta + p^2\delta_{\mu\lambda}\delta_{\alpha\beta}). \tag{2.46e}$$

The equivalent forms of Slavnov–Ward identities (equation (2.43)) in terms of T_i ($i = 1, \ldots, 5$) are

$$T_3 + p^2 T_5 = 0 \tag{2.47}$$

$$(p^2)^2 T_1 + 4(\omega-1)^2 T_2 + 4(\omega-1)(T_3 - p^2 T_4) = 0. \tag{2.48}$$

These identities are satisfied directly from equations (2.40*a*)–(2.40*e*).

3. Ambiguities

Let us consider the transformations

$$I_i(\omega) \to \bar{I}_i(\omega) = I_i(\omega)f_i(\omega) \tag{3.1}$$

where f_i is a real analytic function of ω with $f_i(2) = 1$.

In the previous section the above transformations lead to a set of transformations on equations (2.20)–(2.23), namely

$$I_1 \to f_i(\omega)I_1$$

and

$$\begin{aligned} &\text{equation (2.20)} \to f_2(\omega) \times \text{equation (2.20)}\\ &\text{equation (2.21)} \to f_3(\omega) \times \text{equation (2.21)}\\ &\text{equation (2.22)} \to f_4(\omega) \times \text{equation (2.22)}\\ &\text{equation (2.23)} \to f_5(\omega) \times \text{equation (2.23)}. \end{aligned} \tag{3.2}$$

This set of transformations implies

$$\bar{I} = f_1(\omega)I_1, \qquad \bar{I}_2 = f_2(\omega)I_2, \qquad \bar{I}_3 = f_3(\omega)I_3, \qquad \bar{I}_4 = f_3(\omega)I_4,$$
$$\bar{I}_5 = f_4(\omega)I_5, \qquad \bar{I}_6 = f_4(\omega)I_6, \qquad \bar{I}_7 = f_5(\omega)I_7, \qquad \bar{I}_8 = f_5(\omega), \qquad \bar{I}_9 = f_5(\omega)I_9. \tag{3.3}$$

The new fictitious particle closed-loop contribution$\bar{F}_{\alpha\beta\alpha'\beta'}$ will be

$$\begin{aligned}\bar{F}_{\alpha\beta\alpha'\beta'}(p) = & -k^2\{f_2 p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_2 + (2\omega-1)f_3[p_\alpha p_{\alpha'}\delta_{\beta\beta'}I_3 + p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_4] \\ & -(2\omega+1)f_4[p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_5 + p_{(\alpha}E_{\beta)\alpha'\beta'}I_6] - (2\omega-1)f_4[p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_5 \\ & \times p_{(\alpha'}E_{\beta')\alpha\beta}I_6] - p_{\alpha'}p_{\beta'}f_3[\delta_{\alpha\beta}I_3 + p_\alpha p_\beta I_4] + f_2 p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_2 \\ & + 2\omega f_5[p_\alpha p_\beta p_{\alpha'} p_{\beta'} I_7 + G_{\alpha\beta\alpha'\beta'}I_8 + H_{\alpha\beta\alpha'\beta'}I_9]\}.\end{aligned} \tag{3.4}$$

This should be written in terms of coefficients $\bar{F}_i(p^2)(i = 1, \ldots, 5)$ ie,

$$\begin{aligned}\bar{F}_{\alpha\beta\alpha'\beta'}(p) = & +K^2[p_\alpha p_\beta p_{\alpha'} p_{\beta'} \bar{F}_1(p^2) + \delta_{\alpha\beta}\delta_{\alpha'\beta'}\bar{F}_2(p^2) + (\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \delta_{\beta\alpha'}\delta_{\alpha\beta'})\bar{F}_3(p^2) \\ & + (\delta_{\alpha\beta}p_{\alpha'}p_{\beta'} + \delta_{\alpha'\beta'}p_\alpha p_\beta)\bar{F}_4(p^2) + (\delta_{\alpha\alpha'}p_\beta p_{\beta'} + \delta_{\beta\alpha'}p_\alpha p_{\beta'} + \delta_{\beta\alpha'}p_\alpha p_{\beta'} + \delta_{\alpha\beta'}p_\beta p_{\alpha'} \\ & + \delta_{\beta\beta'}p_\alpha p_{\alpha'})\bar{F}_5(p^2)].\end{aligned} \tag{3.5}$$

The symmetries between indices $\alpha\beta$, α', β' of the second and the fourth terms imply that

$$f_3(\omega) = f_4(\omega) = f_5(\omega) \tag{3.6}$$

so we are left with three arbitary functions f_1, f_2, f_3.

The coefficients $\bar{F}_i(p^2)$ are

$$\bar{F}_1 = -I_1\left(f_2 + f_3\frac{(\omega^3 - 5\omega^2 - 2\omega)}{2(4\omega^2-1)}\right) \tag{3.7a}$$

$$\bar{F}_2 = -\frac{\omega\,.\,(p^2)^2 f_3}{8(4\omega^2-1)}I_1 = \bar{F}_3 \tag{3.7b}$$

$$\bar{F}_4 = -\frac{(2\omega^2+2\omega+1)p^2 I_1 f_3}{8(4\omega^2-1)} \tag{3.7c}$$

$$\bar{F}_5 = -\frac{p^2 I_1 f_3}{16(4\omega^2-1)}. \tag{3.7d}$$

Similar transformations are carried out for the graviton closed-loop contribution and the new function $\bar{R}_{\alpha\beta\alpha'\beta'}(p^2)$ is obtained

$$\begin{aligned}\bar{R}_{\alpha\beta\alpha'\beta'}(p) = & K^2[p_\alpha p_\beta p_{\alpha'} p_{\beta'} \bar{R}_1(p^2) + \delta_{\alpha\beta}\delta_{\alpha'\beta'}\bar{R}_2(p^2) + (\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \delta_{\beta\alpha'}\delta_{\alpha\beta'})\bar{R}_3(p^2) \\ & + (\delta_{\alpha\beta}p_{\alpha'}p_{\beta'} + \delta_{\alpha'\beta'}p_\alpha p_\beta)\bar{R}_4(p^2) + (\delta_{\alpha\alpha'}p_\beta p_{\beta'} + \delta_{\beta\alpha'}p_\alpha p_{\beta'} + \delta_{\alpha\beta'}p_\beta p_{\alpha'} \\ & + \delta_{\beta\beta'}p_\alpha p_{\alpha'})\bar{R}_5(p^2)]\end{aligned} \tag{3.8}$$

where $f_3(\omega) = f_4(\omega) = f_5(\omega)$ because of the symmetry properties of the indices and

$$\bar{R}_1 = \frac{16}{32}I_1\left(4f_1 - 2f_2 + \frac{f_3}{4(2\omega-1)}(\omega^3 - \omega^2 + 8\omega)\right) \tag{3.9a}$$

$$\bar{R}_2 = \frac{(p^2)^2 I_1}{32}\frac{(-7\omega^3 + 2\omega^2 + 13)}{(\omega-1)^2(2\omega-1)} \tag{3.9b}$$

$$\bar{R}_3 = \frac{(p^2)^2 I_1}{32}\left(\frac{4(4\omega^2-5)}{\omega-1}f_1 + \frac{2(-8\omega^2+12\omega-8}{(\omega-1)}f_2 + \frac{(8\omega^3-51\omega^2+83\omega-28)}{(2\omega-1)(\omega-1)}f_3\right) \quad (3.9c)$$

$$\bar{R}_4 = \frac{p^2 I_1}{32}\left(\frac{32}{\omega-1}f_1 + \frac{4(2\omega^2-\omega-6)}{\omega-1}f_2 + \frac{(-14\omega^3+14\omega^2+12)}{(\omega-1)(2\omega-1)}f_3\right) \quad (3.9d)$$

$$\bar{R}_5 = p^2 I_1\left(f_3\frac{(8\omega^2+3\omega+2)}{32(\omega-1)} - 8(\omega+1)f_1\right). \quad (3.9e)$$

The total closed-loop contribution can be obtained by adding equations (3.5) and (3.8) ie,

$$\begin{aligned}\bar{T}_{\alpha\beta\alpha'\beta'}(p) = K^2[&p_\alpha p_\beta p_{\alpha'} p_{\beta'}\bar{T}_1(p^2) + \delta_{\alpha\beta}\delta_{\alpha'\beta'}\bar{T}_2(p^2) + (\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \delta_{\beta\alpha'}\delta_{\alpha\beta'})\bar{T}_3(p^2)\\ &+ (\delta_{\alpha\beta}p_{\alpha'}p_{\beta'} + \delta_{\alpha'\beta'}p_\alpha p_\beta)\bar{T}_4(p^2) + (\delta_{\alpha\alpha'}p_\beta p_{\beta'} + \delta_{\beta\alpha'}p_\alpha p_{\beta'} + \delta_{\alpha\beta'}p_\beta p_{\alpha'}\\ &+ \delta_{\beta\beta'}p_\alpha p_{\alpha'})\bar{T}_5(p^2)]\end{aligned} \quad (3.10)$$

where

$$\bar{T}_1 = I_1\left(2f_1 - 2f_2 + \frac{(2\omega^4-5\omega^3+35\omega^2+16\omega)}{8(4\omega^2-1)}f_3\right) \quad (3.11a)$$

$$\bar{T}_2 = \frac{I_1(p^2)^2}{32}\left(\frac{4(-\omega^2-6\omega+9)}{(\omega-1)^2}f_1 + \frac{4(5\omega-6)}{(\omega-1)}f_2 + \frac{(2\omega^4+9\omega^3-16\omega^2+5\omega+12)}{(\omega-1)^2(4\omega^2-1)}f_3\right) \quad (3.11b)$$

$$\begin{aligned}\bar{T}_3 = \frac{(p^2)^2 I_1}{32}\Bigg[&\frac{4(4\omega^2-5)}{(\omega-1)}f_1 + \frac{2(-8\omega^2+12\omega-8)}{(\omega-1)}f_2\\ &+\left(\frac{8\omega^3-51\omega^2+83\omega-28}{(2\omega-1)(\omega-1)} - \frac{4\omega}{4\omega^2-1}\right)f_3\Bigg]\end{aligned} \quad (3.11c)$$

$$\bar{T}_4 = \frac{p^2 I_1}{32}\left(\frac{32}{\omega-1}f_1 + \frac{4(2\omega^2-\omega-6)}{(\omega-1)} + \frac{(-28\omega^4+6\omega^3+14\omega^2+28\omega+16)}{(\omega-1)(4\omega^2-1)}\right) \quad (3.11d)$$

$$\bar{T}_5 = \frac{p^2 I_1}{32}\left(\frac{-2f_3}{(4\omega^2-1)} + \frac{(8\omega^2+3\omega+2)}{(2\omega-1)}f_3 - 8(\omega+1)f_1\right). \quad (3.11e)$$

The Slavnov–Ward identities are:

$$\bar{T}_3 + p^2\bar{T}_5 = 0 \quad (3.12)$$

$$p^2\bar{T}_1 + 4(\omega-1)^2\bar{T}_2 + 4(\omega-1)(\bar{T}_3 - p^2\bar{T}_4) = 0. \quad (3.13)$$

In order to satisfy the identities (3.12) and (3.13) we must have

$$\text{equation (3.12)} = [(2\omega^2-3)f_1 + 2(-2\omega^2+3\omega-2)f_2 + (2\omega^2-6\omega+7)f_3]\frac{(p^2)^2 I_1}{32(\omega-1)} = 0 \quad (3.14)$$

$$\begin{aligned}\text{equation (3.13)} = (p^2)^2 I_1\Bigg(&\frac{1}{2}(3\omega^2-6\omega)f_1 + (-3\omega^2+6\omega-4)f_2\\ &+\frac{(48\omega^4-96\omega^3+116\omega^2+24\omega-32)}{8(4\omega^2-1)}f_3\Bigg) = 0\end{aligned} \quad (3.15)$$

which give a set of simultaneous equations with three unknowns f_1, f_2 and f_3

$$(2\omega^2-3)f_1+2(-2\omega^2+3\omega-2)f_2+(2\omega^2-6\omega+7)f_3 = 0 \tag{3.16}$$

$$(3\omega^2-6\omega)f_1+2(-3\omega^2+6\omega-4)f_2+(3\omega^2-6\omega+8)f_3 = 0. \tag{3.17}$$

The only possible solution of this set of equations is

$$f_1 = f_2 = f_3. \tag{3.18}$$

We are left with one arbitrary constant $f(\omega)$.

4. Conclusion

In the previous section we have shown, that the transformation (3.1) introduces arbitrary functions $f_i(\omega)$. By applying Slavnov–Ward identities we are left with at least one arbitary function $f(\omega)$. The contribution of $f'(2)$ at the one-loop level to the finite part of the connected Green function and to the Lagrangian counter term (ΔL) is the single ambiguity.

We may encounter more ambiguities as we go to higher-order loops, since the number of arbitrary constants, $f'(2)$, $f''(2)$, etc increases with the order of the loops. To explain this clearly consider a typical integral involved in the one-loop and the two-loop diagrams, namely

$$\bar{I}_1 = \int d^{2\omega}q[q^2(k-q)^2]^{-1} \propto \Gamma(2-\omega) \tag{4.1}$$

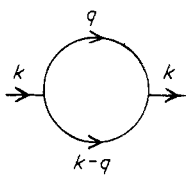

Figure 6.

and

$$\bar{I}_2 = \iint d^{2\omega}q\, d^{2\omega}p[p^2q^2(q-p)^2(p-k)^2(k-q)^2]^{-1} \propto [\Gamma(2-\omega)]^2. \tag{4.2}$$

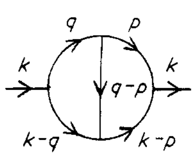

Figure 7.

In the first case, the coefficients of amplitudes T_j can be written as

$$T_j = \Gamma(2-\omega)(p^2)^{\omega-2}f_i(\omega) \qquad j = 1, \ldots, 5. \tag{4.3}$$

Expanding each T_j about $\omega = 2$ gives

$$T_j = \frac{f_i(2)}{(2-\omega)} + [\psi(1)f_j(2) - f_j(2)\lg p^2 - f_j'(2)] + O[(\omega-2)^2] \tag{4.4}$$

where $\lg p^2$ is continued analytically from Euclidean to Minkowski space and $\psi(\omega) = d/d\omega(\lg \Gamma(\omega))$.

Then

$$T_j = \frac{f_j(2)}{(2-\omega)} + [\psi(1)f_j(2) - f_j(2)\lg(p^2) - f_j'(2)] + i\pi f_j(2) + 0[(2-\omega)]^2 \qquad j = 1, 2, \ldots, 5. \tag{4.5}$$

So far for the lowest order the first derivative of $f_j(\omega)$ ie, $f_j'(2)$ contributes to the finite part of single loop.

Similarly for the higher order (eg double loop) the coefficients of amplitude T_j are written as

$$T_j = \frac{f_j(2)}{(2-\omega)^2} + \frac{[\psi(1)f_j(2) - f_j(2)\lg p^2 - f_i(2)]}{(\omega-2)} + [\psi(1)f_j(2) - f_j(2)(\lg p^2)^2 - f_j''(2)] + O(\omega-2)^3. \tag{4.6}$$

So the second derivative of $f_j(\omega)$ ie, $f_j''(2)$ contributes to the finite part of the double loop and consequently contributes to the finite part of connected Green functions and Lagrangian counter terms (ΔL).

Naturally enough all we have said here has been conjectural, since no calculations have been done at the two-loop level. However we do expect these results to have some general validity. There are three points that should be made here that are relevant. One is that if a similar procedure is applied to the fourth-order contribution to the vacuum polarization in QED there is not the same freedom of introducing further ambiguities along the lines we have suggested. The gauge invariance of the theory is liable to be destroyed and the delicate cancellations of single-loop singularities of double-loop integrals would not occur. However that QED is not as ambiguous as we suggest may well be irrelevant to the theory we are discussing. The former theory has very different high-energy behaviour from quantum gravity, so that the two theories may well be completely incomparable.

As of yet no non-trivial double-loop quantum gravity calculation has been made. But there is still the problem of ambiguities in higher loops, which may be different from that at the lower loop level. All we can say is what happens in the latter case; it is at least a prior indication that further ambiguities are present than had been previously realized.

The third aspect of (4.6) which should be noticed is that there is a single-pole term with residue proportional to $\lg p^2$. This term may be cancelled by other terms, especially those arising from the single-loop counter-term resubstituted into the single-loop term. If it is not, then it can only be removed by an appropriate counter-term involving the unpleasant factor $\log \Box^2$. This is a difficulty present independently of the ambiguity we have mentioned, since even if $f(\omega) \equiv 1$ this awkward single-pole term is still present.

There is a further ambiguity which arises as follows. If the integrals $I_2, \ldots, I_9$ are evaluated 'naively' then they are only valid in non-overlapping regions of the ω plane. For instance, integrals I_7, I_8, I_9 are defined in the non-overlapping ranges $1 < \text{Re}\,\omega < 2$, $0 < \text{Re}\,\omega < 1$ and $-1 < \text{Re}\,\omega < 0$ respectively. If regularization is therefore performed in the 'naive' way there is no *unique* analytic continuation in the neighbourhood of $\omega = 2$ and even then it is impossible to evaluate the physical amplitude *unambiguously*.

It was suggested (Capper and Leibbrandt 1972) that an analytic continuation is indeed possible provided equation (2.17) is replaced by the definition

$$\int d^{2\omega}q \exp(-aq^2+2b\,.\,q) \equiv \left(\frac{\pi}{a}\right)^{\omega} \exp\left(\frac{b^2}{a}-af(\omega)\right) \qquad a>0 \tag{4.7}$$

where $f(\omega)$ is a nonzero analytic function.

The ambiguities are still present in the above mentioned approach. First, in order to get equation (2.20) we differentiate (4.7) once with respect to a or twice with respect to b_μ, giving two different results which are not consistent. The consistency of the definition (2.17) has thus been removed by the new definition (4.7). Secondly, the final result will not be independent of the exact form of $f(\omega)$ as has been suggested by Capper and Leibbrandt (1972) by placing reasonable conditions on $f(\omega)$.

We conclude that this second ambiguity is not removable in the suggested manner (Capper and Leibbrandt 1972).

Finally we refer the reader to work of D M Capper and M J Duff on trace anomalies in dimensional regularization.

The ambiguities which arise when the dimensional regularization techniques are applied in background field method to evaluate the one-loop correction of gravity–matter interaction have been discussed by Nouri-Moghadam and Taylor (1974).

Acknowledgment

We would like to thank Mr P Pandya for checking the results.

References

Arnowitt R and Deser S 1959 *Phys. Rev.* **113** 745–50
Arnowitt R, Deser S and Misner C W 1959 *Phys. Rev.* **116** 1322–30
—— 1960 *Phys. Rev.* **117** 1595–602
Ashmore J F 1972 *Nuovo Cim.* **4** 289–91
—— 1973 *Commun. Math. Phys.* **29** 177–89
Capper D M and Duff M J *Imperial College preprint* ICTP/73/10
Capper D M and Leibbrandt 1972 *ICTP Trieste preprint* IC/72/158
—— 1973 *Nuovo Cim.* **6** 117–9
Capper D M, Leibbrandt G and Ramon-Medrano M 1973 *ICTP Trieste preprint* IC/73/26
Capper D M and Ramon-Medrano M 1973 *ICTP Trieste preprint* IC/73/32
Feynman R P 1963 *Acta Phys. Polon.* **24** 697–705
Fradkin E S and Tyutin I V 1970 *Phys. Rev.* D **2** 2841–57
Goldberg J N 1958 *Phys. Rev.* **111** 315–20
Mandelstam S 1968a *Phys. Rev.* **175** 1580–603
—— 1968b *Phys. Rev.* **175** 1604–23
Nouri-Moghadam M and Taylor J G 1974 *King's College, London preprint*
'tHooft G and Veltman M 1972 *Nucl. Phys.* B **44** 189–213